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Inverses of Matrices

The $n \times n$ **identity matrix** is a matrix with 1's on the main diagonal and 0's elsewhere. If A is any $n \times n$ matrix and I is the $n \times n$ identity matrix, then $AI = A$ and $IA = A$.

2 × 2 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Two $n \times n$ matrices A and B are **inverses** of each other if their product (in both orders) is the $n \times n$ identity matrix. That is, $AB = I$ and $BA = I$. An $n \times n$ matrix A has an inverse if and only if $\det A \neq 0$. The symbol for the inverse of A is A^{-1}.

The Inverse of a 2 × 2 Matrix

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - cb \neq 0.$$

Identity Elements Addition: 0

Multiplication: 1

Solve: $\boxed{\cancel{\tfrac{1}{4}}}\,4x = \tfrac{1}{4}20$

$x = 5$

Solve: $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} X = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

$$\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$$

$3a - 2c = -2$

$3b - 2d = 4$

$-7a + 5c = 3$

$-7b + 5d = -1$

Find invers

$\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} X = \begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix}$

$\cancel{A^{-1}A}\,X = A^{-1} \cdot B$

$X =$

$A = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

determinant $= ad - bc = 3 \cdot 5 - (-7)(-2)$

$= 15 - 14 = 1$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

$AX = B$

$X = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} -4 & 18 \\ -5 & 25 \end{bmatrix}$

$X = [A^{-1}] * [B]$